SNA: 2024-2025

Q1:

1) F, has one direction
2) F, simple graph has no loops or multiple edges
3) T
4) F
5) F, link-level feature
6) 7) 8) Not required

Q2:

1) Circuit
2) Cycle
3) Cycle graph
4) nb of nodes: $n = 5$

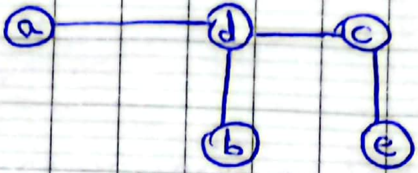
   nb of edges in a complete graph is the nb of all possible edges in the graph:

   $$m = \frac{n(n-1)}{2} = \frac{5(5-1)}{2} = 10$$

5) Degree centrality
6) Eigenvector
7) Clustering coefficient
8) Node-level feature

Q3:

a)



Graph G

b) $dc = \dfrac{\sum a_{ij}}{n-1}$

- $dc(a) = \dfrac{1}{4} = 0.25$
- $dc(b) = \dfrac{1}{4} = 0.25$
- $dc(c) = \dfrac{2}{4} = \dfrac{1}{2} = 0.5$
- $dc(d) = \dfrac{3}{4} = 0.75$
- $dc(e) = \dfrac{1}{4} = 0.25$

→ Node d has the highest centrality based on degree measure (0.75)

c) $cc = \dfrac{N-1}{\sum d(y,x)}$

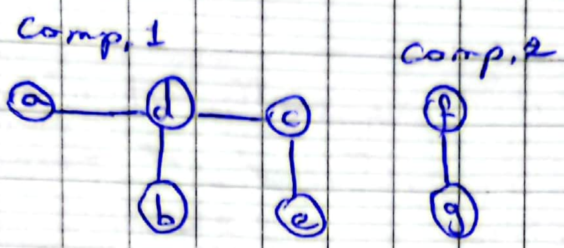
- $cc(a) = \dfrac{4}{2+2+1+3} = \dfrac{4}{8} = 0.5$

where $d(b,a) = 2$; $d(c,a) = 2$; $d(d,a) = 1$, $d(e,a) = 3$

- $cc(b) = \dfrac{4}{2+2+1+3} = 0.5$
- $cc(c) = \dfrac{4}{2+2+1+1} = \dfrac{2}{3} = 0.67$
- $cc(d) = \dfrac{4}{1+1+1+2} = \dfrac{4}{5} = 0.8$
- $cc(e) = \dfrac{4}{3+3+1+2} = \dfrac{4}{9} = 0.44$

→ The Node with highest closeness centrality is d (0.8).

d)



Graph G'

- Graph G' has two components.
- Yes, betweenness centrality can be computed for graph G' since it doesn't require a fully connected graph. Only shortest paths inside each component will contribute.
- Betweenness Calculation:

Binary betweenness matrix:

From	To	Shortest Path	a	b	c	d	e	f	g
a	b	(a, d, b)	0	0	0	1	0	0	0
a	c	(a, d, c)	0	0	0	1	0	0	0
a	d	(a, d)	0	0	0	0	0	0	0
a	e	(a, d, c, e)	0	0	1	1	0	0	0
b	c	(b, d, c)	0	0	0	1	0	0	0
b	d	(b, d)	0	0	0	0	0	0	0
b	e	(b, d, c, e)	0	0	1	1	0	0	0
c	d	(c, d)	0	0	0	0	0	0	0
c	e	(c, e)	0	0	0	0	0	0	0
d	e	(d, c, e)	0	0	1	0	0	0	0
f	g	(f, g)	0	0	0	0	0	0	0
		count	0	0	3	5	0	0	0

with denominator: $\frac{(n-1)(n-2)}{2} = \frac{(5-1)(5-2)}{2} = 6$

then betweenness:

a	b	c	d	e	f	g
$0/6 = 0$	$0/6 = 0$	$3/6 = 0.5$	$5/6 = 0.83$	$0/6 = 0$	$0/6$	$0/6$

→ Node d has the highest betweenness centrality

Q4:

1) To find the clustering coefficient of graph G,
Find the clustering coefficient of every node in G:

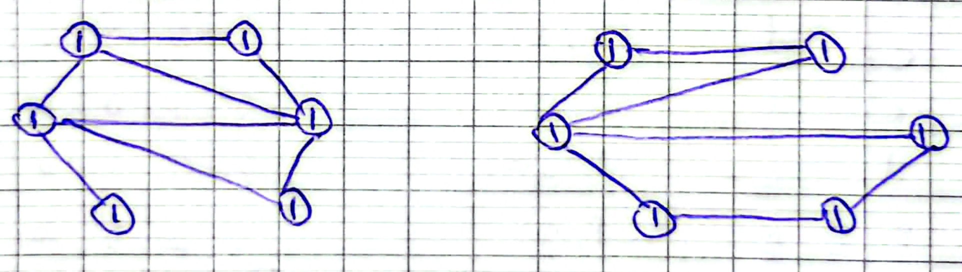
- $cc(a) = \dfrac{2N_a}{dc_a(dc_a - 1)} = \dfrac{2 \times 0}{0.25(0.25-1)} = 0$
- $cc(b) = 0$
- $cc(c) = 0$
- $cc(d) = 0$
- $cc(e) = 0$

$\rightarrow cc(G) = \dfrac{1}{n} \sum_{i=1}^{n} cc(v)$
$= \dfrac{1}{5} [0 + 0 + 0 + 0 + 0] = 0$

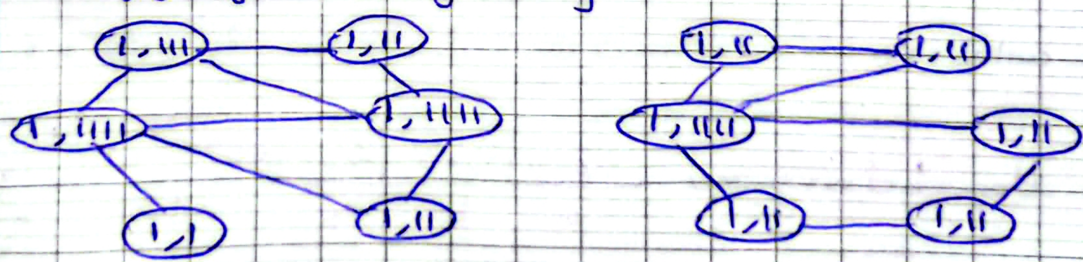
2). Possible nb of edges $= \dfrac{n(n-1)}{2} = 10$
. Possible nb of graphlets: $s = C(4, 10) = 210$
where $s = C(m, k)$; $m =$ nb of current edges of G.

3) Using Weisfeiler-Lehman kernel color refinement algorithm:
- Assign initial color

G^1 G^2



- Aggregate neighboring colors:



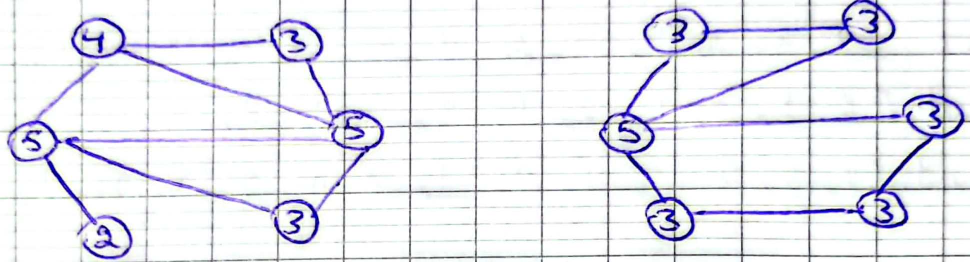
- Hash aggregated colors:

  1,1 → 2

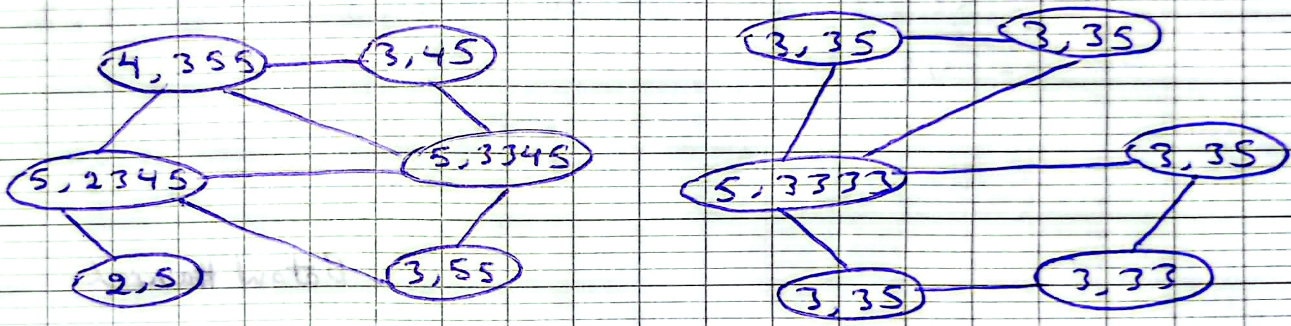
  1,11 → 3

  1,111 → 4

  1,1111 → 5



- Aggregate hashed colors:



- Hash aggregated colors:

  2,5 → 6

  3,33 → 7

  3,35 → 8

  3,45 → 9

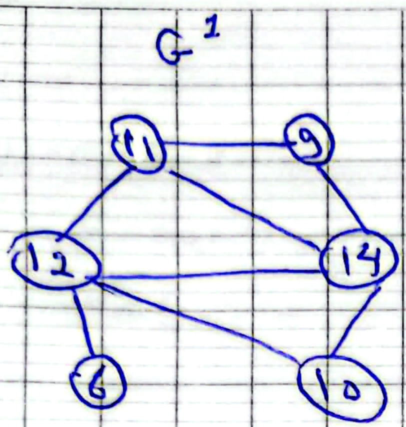
  3,55 → 10

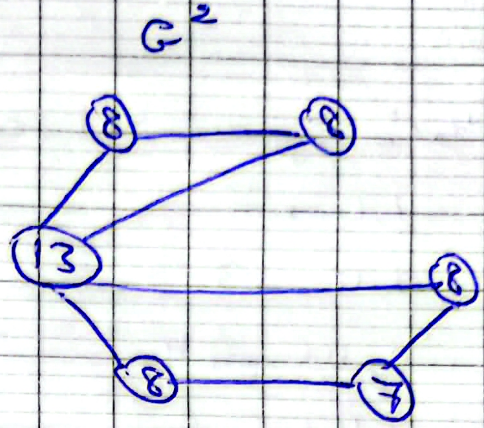
  4,355 → 11

  5,2345 → 12

  5,3333 → 13

  5,3345 → 14





colors : 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14

G^1 counts: [6, 1, 2, 1, 2, 1, 0, 0, 1, 1, 1, 1, 0, 1]

G^2 counts: [6, 0, 5, 0, 1, 0, 1, 4, 0, 0, 0, 0, 1, 0]

$\rightarrow k(G^1, G^2) = \phi(G^1)^T \cdot \phi(G^2)$

$= 36 + 0 + 10 + 0 + 2 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0$

$= 48$